

# Estimation of Optimal International Reserves for Costa Rica: A Micro-Founded Approach

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Research Document No. 01-2012  
Economic Research Department  
February 2012

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The Research Paper Series of the Economic Research Department from Central Bank of Costa Rica in PDF version can be found in [www.bccr.fi.cr](http://www.bccr.fi.cr)

Reference: DEC-DIE-DI-012-2011

## Abstract

In this paper we apply to the economy of Costa Rica a model for the estimation of the optimal amount of international reserves, developed by Jeanne and Rancière (2006, 2011). The main assumption of the model is that the consumers have the possibility to save a part of their income during normal times to smooth the decrease of consumption possibilities during periods of sudden stop crises.

The two most important results are that for all analyzed periods the optimal level of reserves lies above the observed one and that the estimations of optimal reserves show relatively little volatility over time.

Accordingly, the core conclusion of the document is that the Central Bank of Costa Rica should increase its efforts to accumulate international reserves in the medium run.

**Key words:** Optimal International Reserves, Sudden Stops, Financial Dollarization

**JEL classification:** F32, G01, G15

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# Estimation of Optimal International Reserves for Costa Rica: A Micro-Founded Approach

## 1. Introduction

In 2006 The Central Bank of Costa Rica (BCCR, according to its initials in Spanish) began a process with the objective of adopting an inflation targeting regime as their monetary policy framework. Since then, the necessary modeling to better understand the behavior of macroeconomic variables in the country has been developed. This is one of the most important requirements to accomplish a complete implementation of this regime.

One of the most relevant aspects is the estimation of the amount of international reserves that a central bank should accumulate to provide the national economy with stability and to face speculative attacks against the national currency. Currently, the BCCR uses two different ways to determine the amount of reserves that is needed: an ad hoc estimation following the Guidotti rule and an optimization model developed by Muñoz and Tenorio (2010), which is based on a methodology by Ben-Bassat and Gottlieb (1992).

However, the results of this model have recently shown relatively high volatility, and it has become difficult to deduce from them guidelines for policy decisions. For this reason, we develop a different approach to estimate the optimal amount of reserves for Costa Rica. It provides the monetary authority with new information for its policy decisions.

The approach developed is based on a model by Jeanne and Rancière (2006, 2011). It is similar to a model by Gonçalves (2007), that incorporates extensions which take into account the effects of deposit withdrawals and real exchange rate depreciation during a sudden stop. In the Costa Rican case these effects are important, due to the high dollarization of the national economy. Moreover, we incorporate an estimation of a country risk indicator to endogenize the probability of a sudden stop, similar to Ben-Bassat and Gottlieb (1992).

The results obtained for the period from 2005 to 2010 indicate levels of reserves that can be attained by policy makers and are stable over time, if we abstract from the period of crisis that the country suffered between 2008 and 2009.

The document is organized as follows. In section 2 we present a summary of why international reserves are important for emerging economies like Costa Rica and a description of the estimation methods currently used by the BCCR. In section 3 the estimation model for the optimal amount of reserves is developed. The results are described in section 4. Section 5 presents conclusions and recommendations.

## 2. Importance of Reserves and BCCR Indicators

During the last years a continuous increase in the amount of international reserves in emerging countries could be observed<sup>1</sup>. For policy makers in these countries it is important to know if they should follow this trend, because it can be justified on an economic basis or not.

In the literature it is explained that the countries began this process for different reasons. Among the most important ones are the following:

- Insurance against capital flows volatility (Aizenman and Marion, 2003; Stiglitz, 2006)
- Increase in financial globalization (Durdu et al., 2007)
- Prevention of output reduction in the case of a balance of payments deficit (Ben-Bassat and Gottlieb, 1992)
- Export-led growth strategy (Dooley et al., 2003)
- Insurance against the risk of a sudden stop (Jeanne and Rancière, 2009)
- Imitation of the behavior of other economies (Cheung and Qian, 2009)

In the case of Costa Rica two indicators for the recommendable amount of international reserves are currently considered. They are described in the following sections.

### 2.1. Adequate Level of Reserves

The indicators commonly used by monetary authorities for the adequate level of reserves are rules of thumb. There is no precise and unique measure universally considered as adequate. On the contrary, there are a number of different ratios attempting to define a specific benchmark for reserve adequacy.

Among the most cited and empirically supported ones is the ratio of reserves to short-term external debt. This so-called Greenspan-Guidotti rule suggests that the amount of international reserves should equal the amount of external debt with a maturity of one year or less. It is suggested to include the external debt of the private sector, since there is evidence of its excess indebtedness as a consequence of past international crises.

A modification of the Greenspan-Guidotti rule is applied especially in emerging economies. This version also considers a fraction of the broad money supply, weighted by the country risk index. This fraction lies usually between 10% and 20% in economies with a managed float or fixed exchange rate regime and between 5% and 10% in economies using floating exchange rates.

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<sup>1</sup> Muñoz and Tenorio (2010) describe the accumulation of international reserves in emerging countries.

The BCCR currently applies a variation of this method to estimate the adequate level of reserves. The Economic Division of the institution carries out daily calculations using the following equation:

$$R_{ad} = D_{ex} + D_{in} + 15\% \cdot L \cdot CR, \quad (1)$$

where

$R_{ad}$ : Adequate reserves.

$D_{ex}$ : Public and private external debt (including principal and interest) with a maturity of one year or less in dollars

$D_{in}$ : Public internal debt (including principal and interest) of treasury and BCCR bonds with a maturity of one year or less in dollars

$L$ : Balance of the total liquidity of the national banking system in dollars

$CR$ : Country risk index for Costa Rica

## 2.2. Optimal Level of Reserves

As part of the Central Bank's monetary policy migration towards an inflation targeting regime, Muñoz and Tenorio (2010) developed a new framework for estimating the necessary amount of international reserves. This level of reserves is estimated in an optimization process, unlike the adequate level of reserves, which follows discretionary rules.

The optimization process is based on a technique described by Ben-Bassat and Gottlieb (1992). The idea is to consider a monetary authority, which tries to minimize the expected costs of holding international reserves ( $E(C)$ ). These costs consist of the opportunity costs of holding reserves ( $C_1$ ) and its real costs due to a deceleration of economic growth ( $C_0$ ).

The objective function that has to be minimized is:

$$E(C) = \theta C_0 + (1 - \theta)C_1, \quad (2)$$

where  $\theta$  describes the probability of sudden stop crisis as a function of the amount of reserves ( $R$ ).

The probability of a sudden stop crisis can be expressed as a logistic distribution function of a country risk indicator  $f$ :

$$\theta = \frac{\exp f}{1 + \exp f}, \quad (3)$$

which we assume is dependent of the amount of reserves and have the following functional form:

$$f(R) = \alpha_0 + \alpha_1 \ln\left(\frac{R}{M}\right) + \alpha_2 \exp\left(\frac{D}{X}\right) + \alpha_3 \dot{y}, \quad (4)$$

where  $M$  represents the imports,  $D$  the external debt,  $X$  the exports and  $\dot{y}$  the economic growth rate.

The opportunity cost of holding reserves is assumed to be a fraction of the amount of reserves ( $C_1 = rR$ ), where  $r$  is the difference between the interest rate of external debt and the interest yield of reserves. The real costs are assumed to be 15.9% of the GDP, which is an average of the output loss during twin crisis calculated by diverse studies for emergent economies.

The optimal level of reserves is calculated by taking the first derivative of  $E(C)$  with respect to  $R$  and determining the point at which it is zero. As a solution the following optimal amount of reserves ( $R^*$ ) is obtained<sup>2</sup>:

$$R^* = \left( \theta(R^*) \left( \frac{\alpha_1}{R^*} + \frac{\alpha_2}{X} \exp\left(\frac{D}{X}\right) \right) \right)^{-1} + \frac{C_0}{r}. \quad (5)$$

In the case of Costa Rica estimations with this method have been made from 2006 onwards. However, the initial estimations covered only the period until 2009. The recent estimations show higher volatility, which did not appear at the beginning and makes policy decisions based on this measure harder to implement.

### 3. Model

The model that is developed in this paper is based on models by Jeanne and Rancière (2006, 2011) and Gonçalves (2007). However, in this case an extension is added which accounts for the possibility that reserves could help prevent a crisis.

The idea of the model is to consider a small open economy in discrete infinite time, which is exposed to a certain probability of sudden stops. Consumers have the option to save part of their income during normal times to moderate their loss of utility in periods of crisis, so that they can smooth their consumption over time. These sudden stops affect purchasing power in different ways: the diminishment of GDP, the depreciation of the real exchange rate, the impossibility of obtaining external credit for any agent (consumers or government) and capital outflows from the country.

Consumers in a non-crisis period have the possibility of choosing the amount of insurance or reserves that maximizes their expected welfare over time:<sup>3</sup>

$$E(U_t) = E\left(\sum_{i=0}^{\infty} (1+r)^{-i} u(C_{t+i})\right), \quad (6)$$

where  $u(C) = \frac{C^{1-\sigma}-1}{1-\sigma}$  is a utility function with a constant relative risk aversion  $\sigma$  ( $\sigma \geq 0$ ,  $\sigma \neq 1$ );  $r$  is the risk-free interest rate and  $C_t$  the domestic consumption.

<sup>2</sup> It is assumed that external debt is constituted by two differentiable variables: reserves ( $R$ ) and other debt not related with the reserves attained by the country ( $D_n$ ). So we have that  $D = R + D_n$ .

<sup>3</sup> The model does not estimate the optimal level of reserves in periods of crises. However, it is assumed that consumers use their income from the insurance to smooth the effects of the crisis.



Since the level of reserves in period  $t$  only influences the level of consumption in period  $t + 1$ , the optimal amount of reserves can be obtained by maximizing the function:

$$(1 - \theta(\varphi)) u(C_{t+1}^b) + \theta(\varphi) u(C_{t+1}^d), \quad (7)$$

where  $b$  denominates a period prior to the crisis,  $d$  a period of crisis and  $\theta(\varphi)$  is the probability of a sudden stop in the following period.

Unlike the models of Jeanne and Rancière (2006, 2011), and Gonçalves (2007) the probability of crisis is not constant. We assume the same functional form<sup>4</sup> as in the optimal level of reserves measure presented by Muñoz and Tenorio (2010) that depends on a country risk indicator  $f$ :

$$\theta = \frac{\exp f}{1 + \exp f} \quad (8)$$

In contrast with the aforementioned paper, we assume that the country risk indicator has the following form:

$$f(\varphi) = \alpha_0 + \alpha_1 \ln \varphi + \alpha_2 \ln x + \alpha_3 \ln(\lambda_P + \lambda_G) + \alpha_4 \dot{y}, \quad (9)$$

Where  $\varphi$  is the ratio of reserves to output,  $x$  is the ratio of exports of goods to GDP,  $\lambda_P$  is the ratio of private short-term foreign currency debt to GDP,  $\lambda_G$  is the ratio of public short-term foreign currency debt to GDP and  $\dot{y}$  is the Costa Rican economic growth rate.

The consumer is subject to the budget constraint:

$$C_t = Y_t + q_t(B_t - (1 + r_B)B_{t-1} + P_t - (1 + r)P_{t-1} + Z_t), \quad (10)$$

where  $Y_t$  is domestic output,  $q_t$  is the real exchange rate,  $B_t$  is the dollar short-term lending by banks to the private sector,  $P_t$  is the private sector's short-term foreign debt and  $Z_t$  is a transfer from the government.

The international reserves are interpreted as an insurance, which consumers can decide to buy against the negative impacts of crises. In this context the government is seen as an intermediary that can buy foreign debt and securities and transfer it to the consumers. It is assumed that there is always enough insurance offered to cover the optimal amount of reserves on the described conditions.

Consumers pay an insurance premium  $(\delta + \theta(\varphi))R_t$  (where  $\delta$  is the pure risk premium) every period and receives the transfer  $R_t$  in periods of crisis. Besides, the government may issue short-term foreign debt during non-crises periods. This leads to the following transfers before and during a crisis:

$$Z_t^b = G_t - (1 + r)G_{t-1} - (\delta + \theta(\varphi))R_{t-1}, \quad (11)$$

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<sup>4</sup> It is assumed a logistic function because it lies between 0 and 1 at all times and the marginal effect of a change in  $f$  is exponential for a small  $f$  and gets smaller with bigger absolute values of  $f$ .

$$Z_t^d = -(1+r)G_{t-1} + (1-\delta-\theta(\varphi))R_{t-1}. \quad (12)$$

The financial sector in the domestic economy is subject to the following budget constraint:

$$B_t - (1+r_B)B_{t-1} + A_t - (1+r)A_{t-1} = D_t - (1+r)D_{t-1}, \quad (13)$$

where  $A_t$  are the investment of the banks in risk-free short-term foreign assets and  $D_t$  are the short-term foreign currency deposits for which banks pay interest rates.

It is assumed that  $A_t$  is a constant fraction of the short-term foreign currency deposits for which banks pay interest rates ( $A_t = \alpha D_t, 0 < \alpha < 1$ ).

The interest rates  $r_B$  and  $r$  are given and assumed to be constant, and consumers and the government do not default on their short-term foreign debt.

Before the crisis the exchange rate is constant and normalized to one, during the crisis it depreciates by  $\Delta q$ . Furthermore, during periods of crisis a fraction  $\gamma$  of output is lost, and a fraction  $\tau$  of dollar deposits is withdrawn from the banks. Short-term foreign debt of the government and the private sector is not rolled over and is therefore equal to zero.

This leads to the following equations for domestic consumption before and during the crisis:

$$C_t^b = Y_t^b + (1-\alpha)D_t^b + P_t^b + G_t^b - (1+r)\left((1-\alpha)D_{t-1}^b + P_{t-1}^b + G_{t-1}^b\right) - (\delta + \theta(\varphi))R_{t-1}, \quad (14)$$

$$C_t^d = (1-\gamma)Y_t^b + (1+\Delta q)\left((1-\tau)D_{t-1}^b - (1+r)\left((1-\alpha)D_{t-1}^b + P_{t-1}^b + G_{t-1}^b\right) + (1-\delta-\theta(\varphi))R_{t-1}\right). \quad (15)$$

Because the maximization problem only depends on  $\varphi = \frac{R_t}{Y_{t+1}^b}$ , the equivalent problem of maximizing the following function is considered:

$$m(\varphi) := (1-\theta(\varphi))u\left(\frac{C_{t+1}^b}{Y_{t+1}^b}\right) + \theta(\varphi)u\left(\frac{C_{t+1}^d}{Y_{t+1}^b}\right). \quad (16)$$

Assuming that output grows in non-crisis periods at a constant rate  $\dot{y}^p$  ( $Y_{t+1}^b = (1+\dot{y}^p)Y_t^b$ ), the following equations are obtained:

$$\frac{C_{t+1}^b}{Y_{t+1}^b} = 1 + (1-\alpha)\lambda_D + \lambda_P + \lambda_G - \frac{1+r}{1+\dot{y}^p}\left((1-\alpha)\lambda_D + \lambda_P + \lambda_G\right) - (\delta + \theta(\varphi))\varphi, \quad (17)$$

$$\frac{C_{t+1}^d}{Y_{t+1}^b} = 1 - \gamma + (1+\Delta q)\left(\frac{1-\tau}{1+\dot{y}^p}\lambda_D - \frac{1+r}{1+\dot{y}^p}\left((1-\alpha)\lambda_D + \lambda_P + \lambda_G\right) + (1-\delta-\theta(\varphi))\varphi\right). \quad (18)$$

where  $\lambda_i = \frac{i_t^b}{Y_t}$  for  $i = D, P, G$ .

The optimal level of reserves is calculated by taking the first derivative of  $m(\varphi)$  with respect to  $\varphi$  and determining the point at which it is zero:

$$m'(\varphi) = 0 \Leftrightarrow (1 - \theta(\varphi)) u'(C_{t+1}^b) - \theta'(\varphi) u(C_{t+1}^b) + \theta(\varphi) u'(C_{t+1}^d) + \theta'(\varphi) u(C_{t+1}^d) = 0. \quad (19)$$

In contrast to the results obtained by Jeanne and Rancière (2006, 2011) and Gonçalves (2007), the solution to this problem cannot be expressed in a closed form because of the endogenization of the probability of crisis. The ratio of the optimal level of reserves to output  $\varphi^*$  can be expressed as:

$$\varphi^* = \frac{1 + \frac{y^p - r}{1 + y^p} \lambda - p^{1/\sigma} \left( 1 - \gamma + (1 + \Delta q) \left( \frac{1 - \tau}{1 + y^p} \lambda_D - \frac{1 + r}{1 + y^p} \lambda \right) \right)}{p^{1/\sigma} (1 + \Delta q) (1 - \delta - \theta(\varphi^*)) + \delta + \theta(\varphi^*)}, \quad (20)$$

where  $\lambda = (1 - \alpha)\lambda_D + \lambda_P + \lambda_G$  and

$$p = \frac{u'(C_{t+1}^d)}{u'(C_{t+1}^b)} = \frac{(1 - \theta(\varphi))(1 - \sigma)(\theta'(\varphi)\varphi + \delta + \theta(\varphi)) + \theta'(\varphi)C_{t+1}^b}{(1 + \Delta q)\theta(\varphi)(1 - \sigma)(1 - \delta - \theta(\varphi) - \theta'(\varphi)\varphi) + \theta'(\varphi)C_{t+1}^d}. \quad (21)$$

This result shows that an increase in  $\lambda$ ,  $\gamma$  or  $\tau$  implies an increase in the amount of reserves that is considered optimal. For any of the other parameters it is difficult to determine the direction of their effect on the optimal level of reserves. Moreover,  $p$  can be interpreted as the consumer price of a “sudden stop dollar” in terms of a “normal times dollar”. In this regard, the model shows similar results as those presented by Jeanne and Rancière (2006, 2011) and Gonçalves (2007).

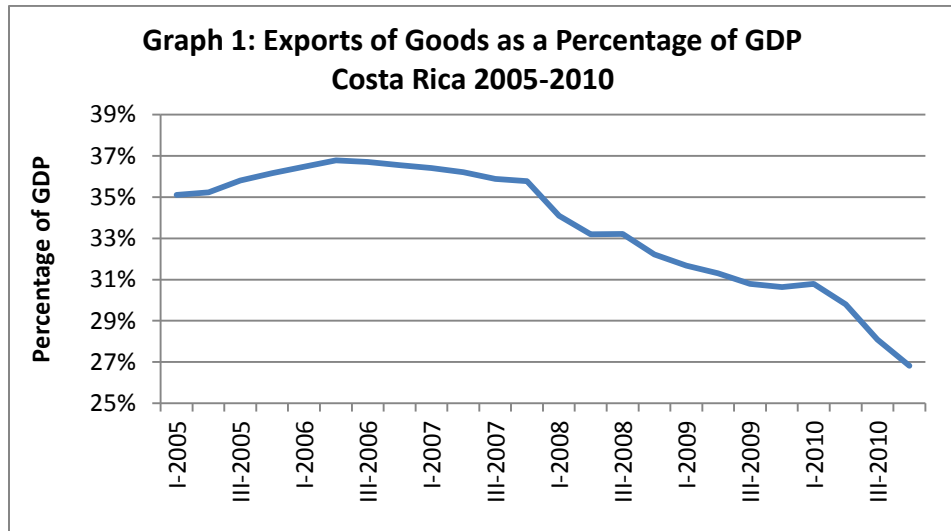
## 4. Estimation

In this section the estimation process and the main results are presented. First, we describe the data we used for the estimations. In a second part, the result of the estimation of the country risk indicator is presented. The last two sections summarize the results of the optimization process and describe how the results change with the modification of some parameter values.

### 4.1. Data

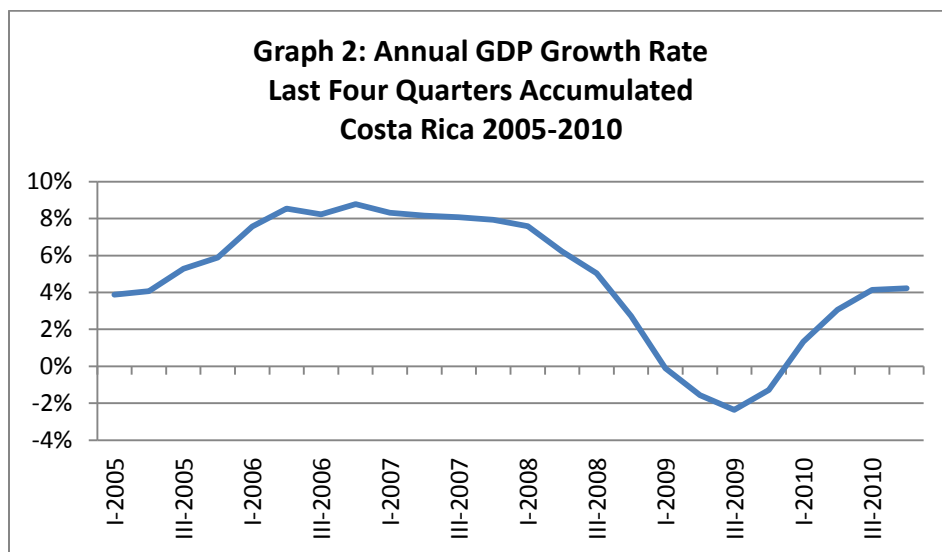
To carry out the estimations it is necessary to obtain local information, which is compatible with the definitions given in the model. We use quarterly series in the period from 2005 to 2010 for exports of goods, real GDP growth, international reserves, public and private debt with a maturity of one year or less, M2 money supply in foreign currency and the total reserves that the Costa Rican financial system holds abroad. All of these series are created by the Department of Macroeconomic Statistics at the Central Bank of Costa Rica.

As it can be seen in Graph 1, exports of goods as a percentage of GDP in Costa Rica were relatively stable until the crisis. Since 2008, there has been a decline worldwide trade, which also affected the economy of Costa Rica. In the near future, it can be expected that the decreasing trend stops and that the volumes of trade increase again. In the context of this work, this implies a risk indicator that is higher than in the past, which could lead to an increase in the need of foreign reserves.



Source: BCCR and own estimation

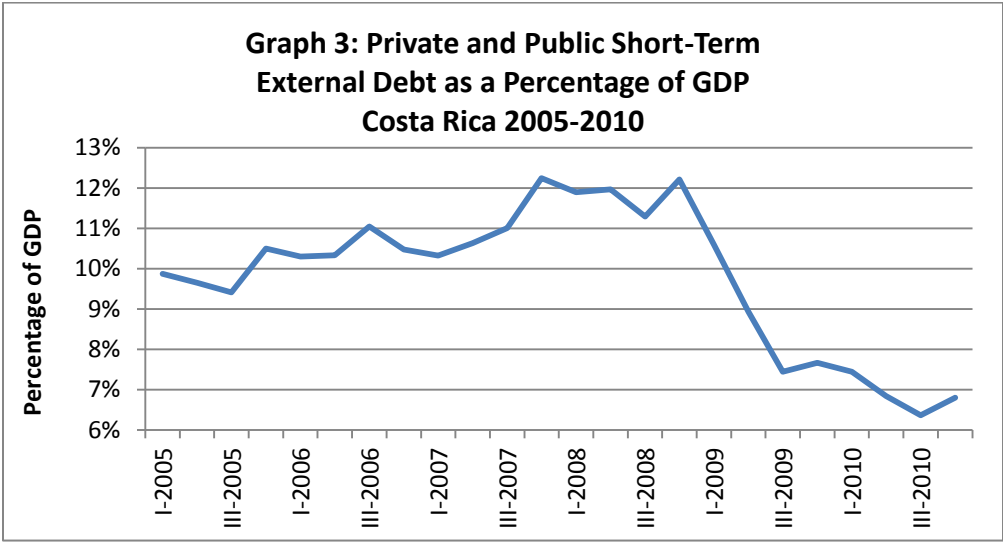
With respect to the growth rates of GDP we are studying a period with high volatility. In Graph 2 it is shown that the country faced above-average growth rates during 2006 and 2007. These rates can be explained with a real estate boom in touristic areas and the establishment of facilities of important international enterprises in sectors such as life sciences and services in Costa Rica. Then, during 2008 and 2009 Costa Rica confronted a crisis as a result of the one that struck the United States. At its lowest point the economy faced a year-on-year decrease in GDP of around 2%. The year 2010 was a period of recuperation, at the end of which the potential GDP growth rate was almost reached. This volatility in the GDP growth rates could lead to notable volatility in the results of the estimation of the country risk indicator.



Source: BCCR and own estimation

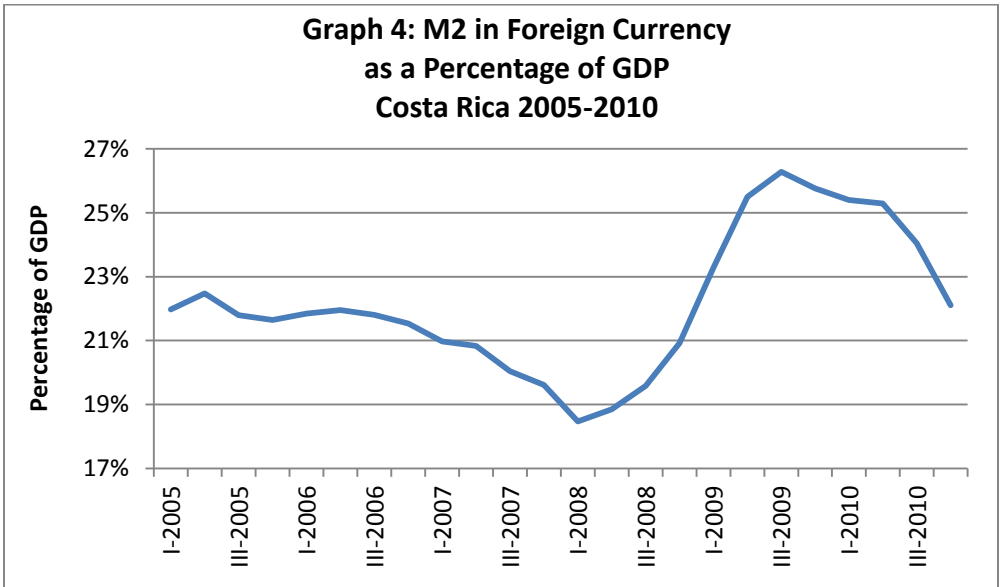
The levels of short-term external debt show an increasing trend until the end of the year 2008. Afterwards, strong restrictions on accumulating short-term debt were imposed due to the international crisis. This affected the capacity of the financial system to obtain new debt during

this period. The decline of short-term debt at the end of the period could imply a decrease in the need of international reserves.



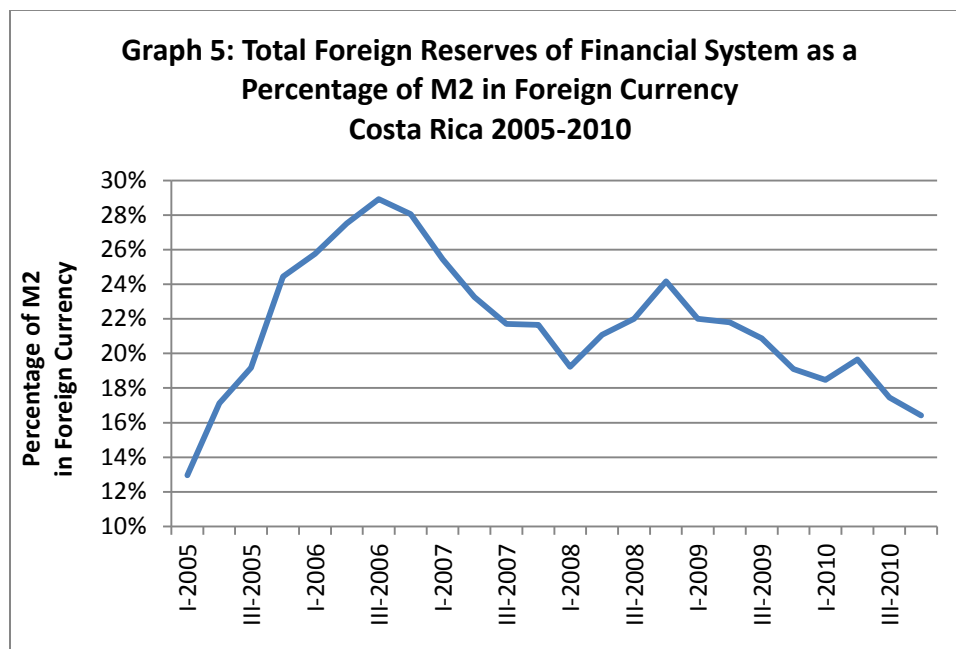
Source: BCCR and own estimation

According to Graph 4, during the first periods the M2 money supply in foreign currency decreased and reached a minimum level of 18.49% of GDP in the first quarter of 2008. This decrease results mainly from the trust of economic agents in the exchange rate stability of the national currency. During this period the nominal exchange rate depreciated about 18% and it was not expected by agents that this trend would continue in the future because it was very close to the threshold at which the Central Bank was compromised to intervene in the market. Afterwards, some turbulence in the international economy and an appreciation of the currency affected this trust and led to an increase in the demand for foreign currency. Due to the recent recovery of the economy and a positive premium granted for investing in national currency, the M2 money supply in foreign currency decreased sharply.



Source: BCCR and own estimation

Finally, the total foreign reserves of the financial system as a percentage of the money supply M2 in foreign currency shows an irregular behavior, as it is stated shown in Graph 5. However, there is a small increase during 2008 due to the international crisis. At the end of the period of analysis, foreign reserves decrease. This implies that the central bank should accumulate more reserves to protect the deposits in foreign currency.



Source: BCCR and own estimation

## 4.2. Country Risk Indicator

We estimate the country risk indicator using an OLS<sup>5</sup> regression. As the dependent variable we take the following proxy variable<sup>6</sup>:

$$f_t = \ln\left(\frac{\Delta i_t - \delta}{1 + r_t^l}\right), \quad (22)$$

where

$\Delta i$  : a measure named SPRDCR20 index, which corresponds to the spread between long term interest rates of Costa Rican bonds and US Treasury bond rates, calculated by the Assets and Liabilities Management Division of the BCCR

$\delta$  : pure risk premium, assumed to be constant and equal to 1.2%

$r^l$  : twelve month London Interbank Offered Rate

<sup>5</sup> Ben-Bassat and Gottlieb (1992) recommend using a 2SLS regression. However, in our case, no difference was found between both estimations and the OLS regression was chosen for simplicity.

<sup>6</sup> According to Jeanne and Rancière (2006, 2011) the interest rate spread that investors charge the BCCR consists of two different parts: the pure risk premium that they charge because of their risk aversion and a charge because of the probability that the country might default on its debt. Accordingly,  $\theta_t = \Delta i_t - \delta$ .

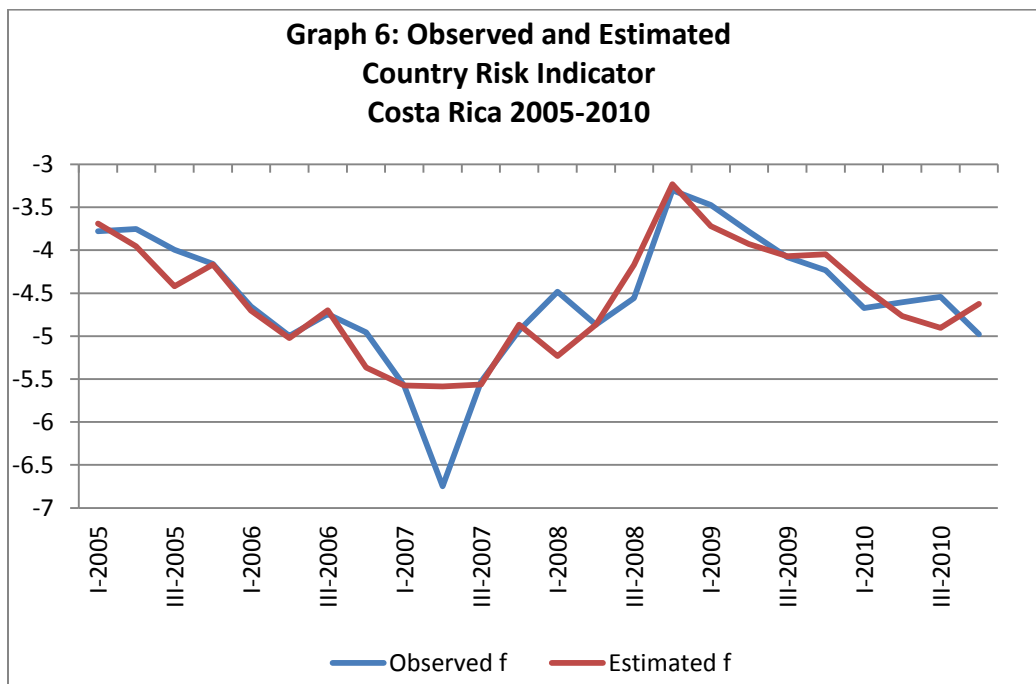
This approximation to the country risk indicator is similar to that of Ben-Bassat and Gottlieb (1992), although we consider only the part of the spread that corresponds to the probability of default in the numerator, according to Jeanne and Rancière (2006, 2011).

When we estimate the regression coefficients, as shown in (9), we obtain results that are statistically different from zero, with coefficients that have the expected signs and where neither autocorrelation nor heteroscedasticity are detected. The results can be expressed in the following function<sup>7</sup> with student's t- statistic in parentheses:

$$f(\varphi) = -8.9748 - 4.5785 \ln(\varphi) - 5.2418 \ln(x) + 4.2425 \ln(\lambda_p + \lambda_G) - 22.3969 \dot{y}. \quad (23)$$

(-4.17)
(-5.75)
(-3.31)
(5.71)
(-5.57)

The proxy series for the country risk indicator and the regression-adjusted series are presented in Graph 6. It can be seen that the adjustment is very good, except for the second quarter of 2007. In this period the spread that the Central Bank of Costa Rica paid was historically low, which cannot be explained by characteristics of Costa Rican economy. One possible explanation is that optimism regarding the global economy was prevalent during that period and crises were not expected, and then agents did not take in account all disposable information in its making decisions process.



Source: Own estimation

<sup>7</sup> The regression has an R-squared of 0.7881. The tests of autocorrelation and heteroscedasticity are presented in the appendix.

### 4.3. Results

Unlike the models of Jeanne and Rancière (2006, 2011) and Gonçalves (2007), the equation for the optimal amount of reserves cannot be solved explicitly. In order to calculate its solution Newton's method was applied.

Starting from an arbitrary  $\varphi_0 \in (0,1)$  the solution to  $m'(\varphi) = 0$  is calculated iteratively by:

$$\varphi_{i+1} = \varphi_i - \frac{m'(\varphi_i)}{m''(\varphi_i)}. \quad (24)$$

The iteration process is stopped as soon as  $\varphi_{k+1} = \varphi_k$  for  $k \in \mathbb{N}$ , as  $\varphi^* := \varphi_{k+1} = \varphi_k$  is a solution to  $m'(\varphi) = 0$ :

$$\varphi^* = \varphi^* - \frac{m'(\varphi^*)}{m''(\varphi^*)} \Leftrightarrow \frac{m'(\varphi^*)}{m''(\varphi^*)} = 0 \Leftrightarrow m'(\varphi^*) = 0. \quad (25)$$

The optimal level of reserves is estimated quarterly for the period from 2005 to 2010 for three different scenarios: a mild crisis, an average crisis and a severe crisis. The three different scenarios vary in the values of the parameters for the GDP loss and the exchange rate depreciation during the crisis.<sup>8</sup>

In all cases the fraction  $\tau$  of dollar deposits that is withdrawn from the banks in crisis is assumed to be 30%, as proposed by Gonçalves (2007) and Magnusson (2011). This value corresponds to the observed withdrawals of residents during the Uruguayan crisis in 2002. Unlike in the model of Gonçalves (2007) the distinction between residents' and non-residents' deposits is not made due to lack of information for Costa Rica. It is assumed though that most of the deposit holders are residents that are imposed strong restrictions and therefore cannot withdraw all their money quickly. Following Jeanne and Rancière (2006, 2011) and Gonçalves (2007) the risk aversion  $\sigma$  is set at 2, a standard value in the study of economic growth and business cycles. The rate  $\dot{y}^p$  at which output grows in non-crisis periods is assumed to be 4.4%, the value typically used by the BCCR for internal estimations. As proposed by Jeanne and Rancière (2006, 2011) and Gonçalves (2007) the risk-free interest rate  $r$  is set at 5%, the approximate average of US 3-month Treasury Bill rates over the last twenty years.<sup>9</sup>

For the average crisis scenario, the fraction  $\gamma$  of output which is assumed to be lost during periods of crisis is taken from Jeanne and Rancière (2006, 2011), the average output loss during the first year of crisis in middle-income economies<sup>10</sup>, which is 6.3%. In the case of a mild crisis it is assumed to be 1% and in a severe crisis 11%. According to what could be observed in Uruguayan

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<sup>8</sup> Intervals for optimal reserves could also be estimated by considering the uncertainty in the estimation of the indicator function  $f$ . However, since we do not get any new information and results are more complicated to interpret we do not add it into the document.

<sup>9</sup> If the average is calculated for the last twenty years, the exact average rate is 3.28%. However, a rate of 5% is used, because that is what is generally found in the literature and does not change the results significantly.

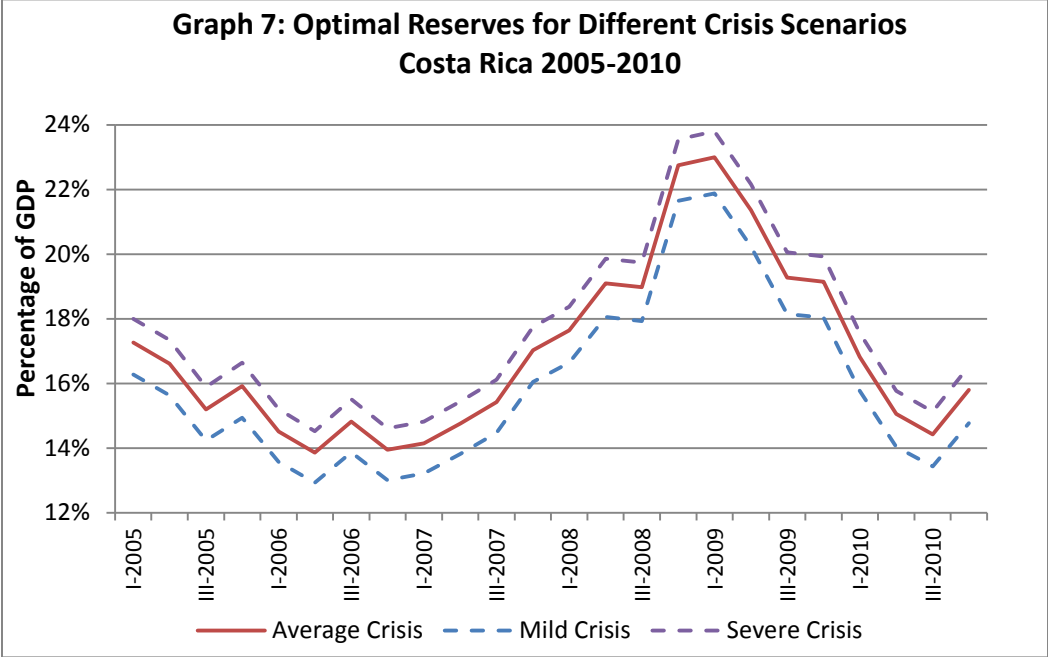
<sup>10</sup> The sample of economies includes the countries classified by the World Bank as "lower-middle-income" and "upper-middle-income" economies, plus Korea and minus major oil-producing economies.



crisis in 2002 the exchange rate depreciation ( $\Delta q$ ) is expected to be 30% in a severe crisis. In a mild crisis no depreciation is assumed and in the case of an average we use a value of 15%.

The average crisis is the type of crisis that is assumed most likely to occur and therefore forms the basis of the BCCR’s estimations. The other scenarios can be seen as variations that are studied in order to get an idea of the effect that different types of crises could have on the results. Besides, it creates an interval that permits policy makers to have a range for their decisions in the future.

Graph 7 shows the optimal level of reserves for the three different scenarios.



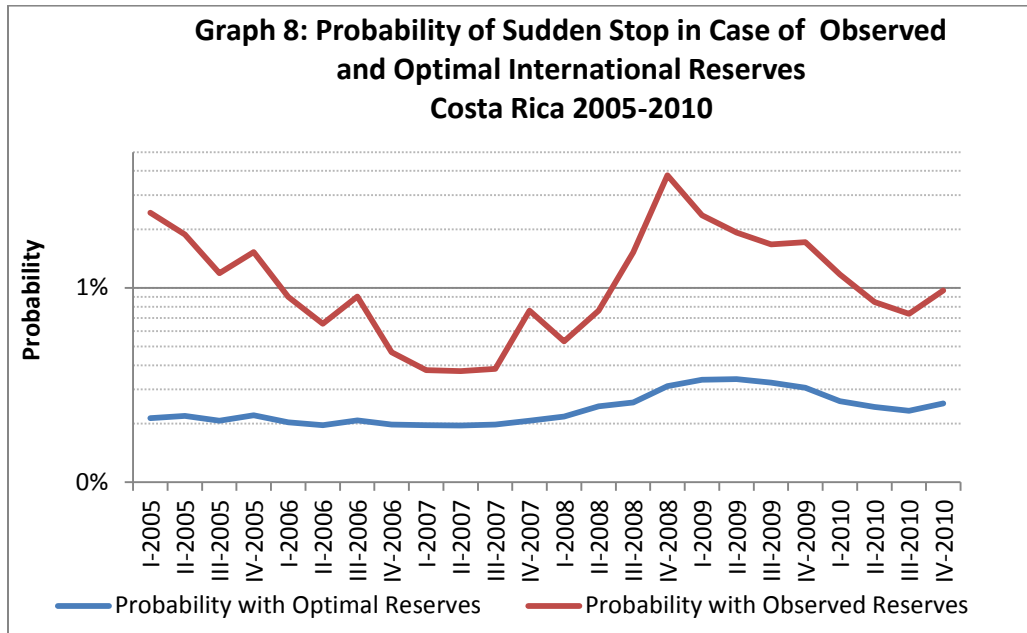
Source: Own estimation

Apart from the years 2008 and 2009 the optimal level of reserves lies between 13% and 18% in all cases, and the results of the different types of crises vary by no more than 2 percentage points.

During the period from 2008 to 2009 the result of optimal reserves is relatively high, up to nearly 24% in the case of the severe crisis. These levels contrast with the amount of reserves that the BCCR actually held in this period (approximately 14% of GDP). However, these results are not surprising. The basic assumption of the model is that the economy is not facing a crisis at the moment, while the goal is to find an optimal level of reserves to prevent crises in the future. Since we know that this was not true in that two year period, the results cannot be considered optimal anymore. On the contrary, the BCCR diminished its reserves during this period to try to counteract the negative effects of the crisis and provide stability to the national financial system. It is therefore natural to exclude these periods in the following calculations and recommendations for policy decisions.

Graph 8 plots the probability of a sudden stop for two different scenarios: one of them considers the observed level of reserves and the other one the optimal level for average crisis

previously estimated. In the graph we use logarithmic scale to provide a better visualization of the data. First, it is important to point that in some of the periods in case of the observed reserves the probability lies below 1% mainly due to a very high economic growth rate. When optimal reserves are considered, the probability of a sudden stop is very low in all periods. It is also worth mentioning that starting in the fourth quarter of 2007 the probabilities in both scenarios rise. This is due to the fact that in these periods it was very costly for agents to save money to prevent an eventual default.



Source: Own estimation

It is important to say that the estimation process in each period is independent of the results obtained for any other period. Accordingly, the average of the estimation results for the periods without crisis could be used as a guideline for the demand of international reserves by national agents. This average is presented in Chart 1. It can be seen in the chart that the BCCR should have accumulated on average an amount of reserves between 14.38% and 16.06% in the period from 2005 to 2010, except for the periods of crisis, and that these estimates have little variability over time.

	Mild Crisis	Average Crisis	Severe Crisis
<b>Average</b>	14.38%	15.35%	16.06%
<b>Standard Deviation</b>	1.09	1.12	1.14
<b>Coefficient of Variation</b>	7.57%	7.29%	7.09%

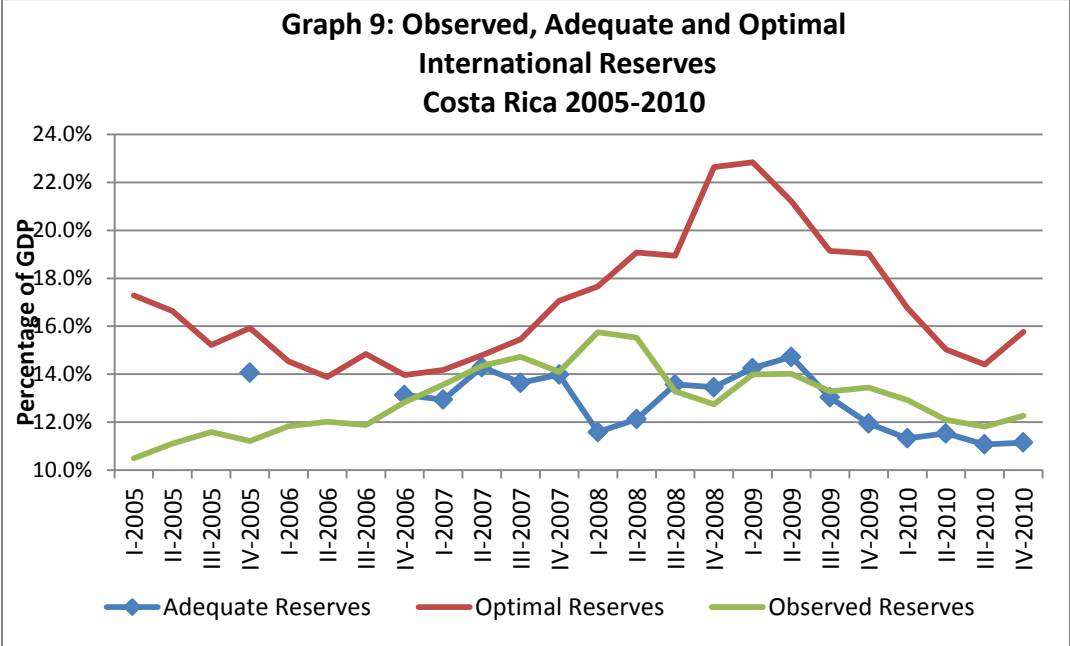
<sup>a</sup>The period from 2008 to 2009 is excluded because it is considered a crisis period.

Source: Own estimation

The optimal reserves estimated are above both the observed reserves and the adequate reserves indicator, as it is shown in Graph 9. It is also worth noting that at the beginning of 2007

the amount held by the Central Bank of Costa Rica was above the lower limit of the optimal interval and very close to the optimal level.

This means that the BCCR should make a strong effort to increase its reserves in the medium run. As a reference, the difference between the estimated optimal amount of reserves and the observed one reaches an average amount of 1.058 million dollars during 2010. Nevertheless, it can also be seen that there existed a similar gap between the adequate reserves indicator and the actual amount of reserves in 2005<sup>11</sup>. The central bank increased its reserves from that moment on, leading to the observed reserves tracking closely the adequate reserves.



Source: Own estimation

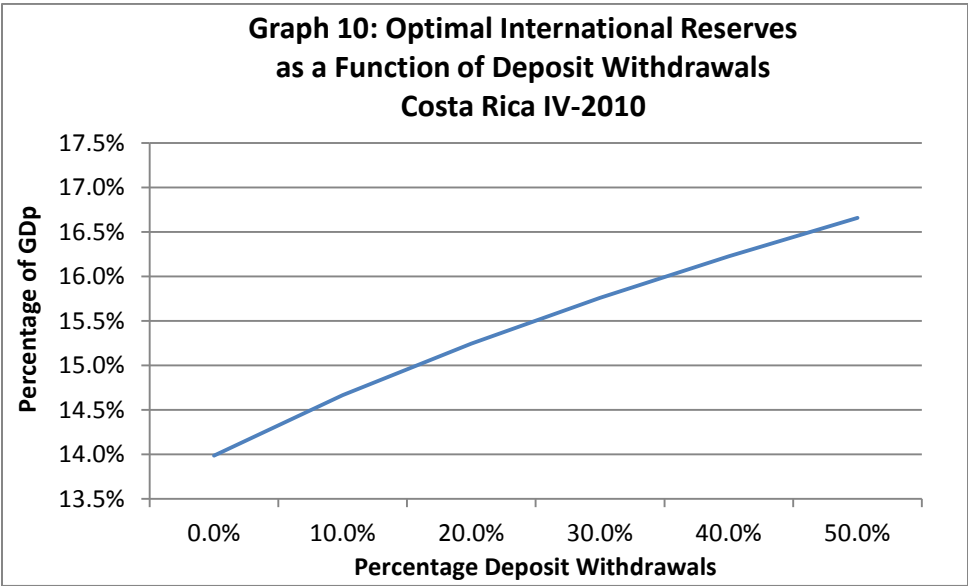
**4.4. Sensitivity Analysis**

In the previous section the optimal level of reserves was estimated for the most realistic choices of parameters. In this section the sensitivity to these choices is put to test assuming an average crisis. For the sensitivity analysis only the results for the fourth quarter of 2010 are presented.

The optimal level of reserves is particularly sensitive to the fraction of dollar deposits, which is withdrawn from the banks and the fraction of output that is lost during a sudden stop.

If it is assumed that 50% instead of 30% of dollar deposits is withdrawn from banks, the optimal level of reserves rises from 15.76% to 16.66%. In case of no withdraw a level of only 14% is optimal. This is consistent with expectations as a withdrawal of money implicates a drop in consumption and therefore a more severe effect of the sudden stop.

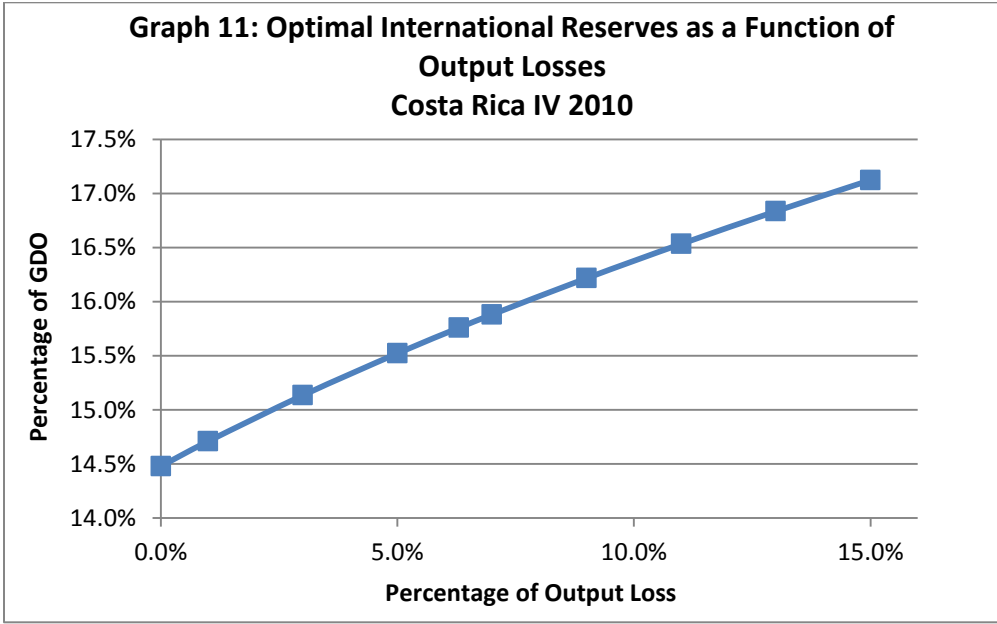
<sup>11</sup> There is no available information for the first three quarters of 2006.



Source: Own estimation

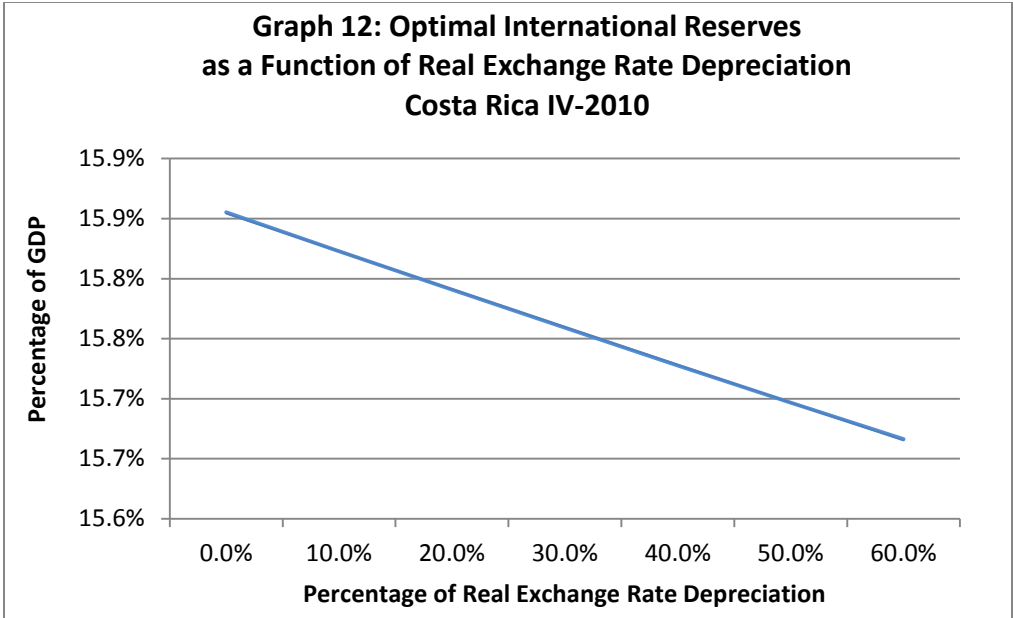
Higher output losses also lead to an increase in the costs of a sudden stop crisis and therefore to a rise in the optimal level of reserves. A variation in the output loss between 0% and 15% leads to optimal levels of reserves between 14.5% and 17% as can be observed in graph 11.

Similar effects have also been observed by Jeanne and Rancière (2006, 2011), Gonçalves (2007), and Magnusson (2011).



Source: Own estimation

The optimal level of reserves is less sensitive to changes in the real depreciation rate. A rate of 60% instead of the originally used 30% causes a decrease in the optimal level of reserves of 0.09 percentage points, whereas if we consider that economy does not suffer any depreciation we obtained an increase of 0.1 percentage points in the estimation of optimal reserves compared to the original level.

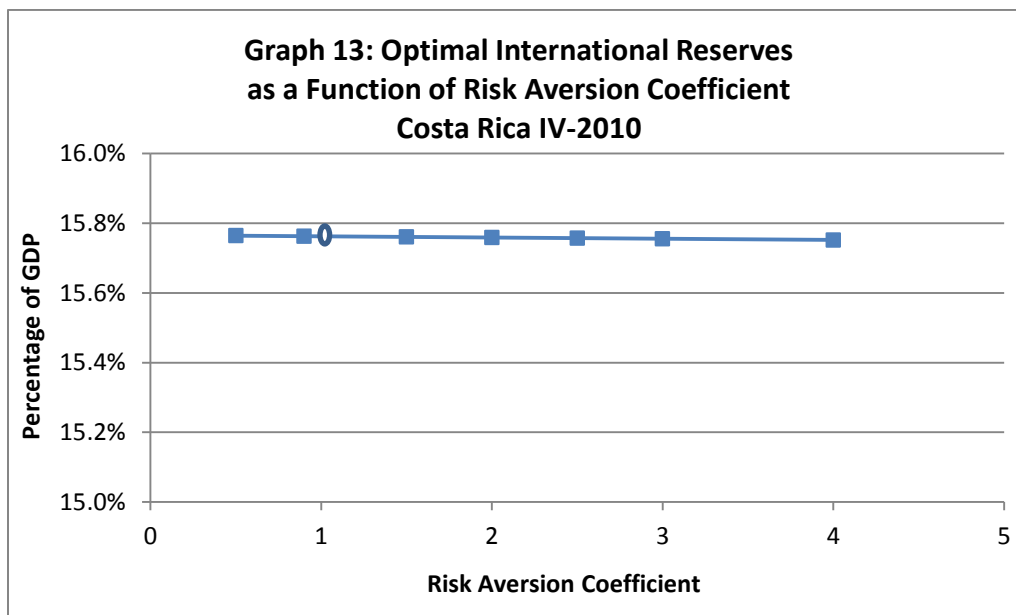


Source: Own estimation

A priori the direction of the effect of a change in the depreciation rate cannot be determined. As it can be seen in equation (18) an increase in the depreciation rate has two effects in the domestic consumption during a crisis: it increases the level of debt, but also the insurance payoff.

For example, Jeanne and Rancière (2006, 2011) and Gonçalves (2007), find the opposite effect of an increase in the depreciation rate due to their assumption of a constant probability of a sudden stop. In contrast, our assumptions imply that the negative effect of the increase of debt is stronger than the positive effect of the rise in the insurance payoff. In the case of an endogenous probability the optimal level of reserves leads to a very small probability of a sudden stop and therefore to a higher insurance payoff compared to the case of a constant probability. For this reason the increase in the insurance payoff exceeds the rise in the debt level.

Changes in the risk aversion coefficient do not lead to significant shifts in the level of reserves. A change from a risk aversion of 0.5 to 4 causes a decline in reserves of 0.01 percentage points.



Source: Own estimation

In Jeanne and Rancière (2006, 2011) and Magnusson (2011) however, a significant sensitivity was found. This can be explained as in the case of the real depreciation rate. Due to the small probability of sudden stop for the optimal level of reserves in the case of an endogenous probability the relevance of risk aversion coefficient becomes insignificant.

## 5. Conclusions and Recommendations

When implementing an inflation targeting regime it becomes necessary to make the exchange rate more flexible. Emerging countries are exposed to speculative attacks and balance of payment crises that can lead to a loss of welfare for the local agents. One way in which central banks can help local agents, is to maintain an adequate level of international reserves that permits the consumers to smooth their consumption path over time.

In this document an application of the model by Jeanne and Rancière (2006, 2011) for the Costa Rican economy was presented. The extensions presented by Gonçalves (2007) were implemented and a strategy to consider that maintaining reserves diminishes the probability of a sudden stop in the future was developed. This strategy is based on the estimation presented by Ben-Bassat and Gottlieb (1992).

The estimation of the optimal amount of reserves was presented for the period between 2005 and 2010. It could be seen that the estimated optimal reserves lie above the observed reserves for the Costa Rican case during this period. Besides, when we abstracted from the period of the international financial crisis (2008-2009), we found that the average amount of reserves maintained by the BCCR should have been 15.35%. In addition, the results showed little volatility over time when we excluded the same period.

The results were presented for three different scenarios that represent different kinds of crises. The idea was to create an interval, which can be used by the Central Bank as a reference for the acquisition of international reserves. For all periods the different scenarios create an interval of nearly two percentage points for the international reserves that the BCCR should hold. This allows the monetary authority to decrease the volatility in the acquisition of reserves shown by the model.

According to this, the BCCR should try to increase its international reserves in the medium term and reach the level that is considered optimal. In fact, at the beginning of 2007 the observed reserves lay above the lower limit of the optimal level.

Besides accumulating international reserves, the BCCR could implement other measures to foster a decrease of the optimal level of reserves in the future. Two of them are to discourage the dollarization of the local financial system and the acquisition of short-term external debt.

Other factors not controlled by the Central Bank that could lower the probability of sudden stop crisis in the future are an increase in the level of exports or economic growth on par with its potential rate.

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## 7. Appendix

### 7.1. Test Results for Country Risk Indicator Regression

#### Chart 2: Heteroscedasticity Test: White's Test

F-statistic	0,541106	Prob. F(14,10)	0,8576
Obs*R-squared	10,77564	Prob. Chi-Square(14)	0,7036
Scaled explained SS	18,69309	Prob. Chi-Square(14)	0,1770

Dependent Variable: RESID^2

Method: Least Squares

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	213,4967	273,4170	0,780847	0,4530
Ln( $\phi$ )	157,5853	202,4473	0,778401	0,4544
(Ln( $\phi$ ))^2	18,02109	14,94945	1,205468	0,2558
(Ln( $\phi$ ))*(Ln( $\lambda_G + \lambda_p$ ))	-4,954524	30,51625	-0,162357	0,8743
(Ln( $\phi$ ))* $\dot{y}$	-196,2188	274,2683	-0,715427	0,4907
(Ln( $\phi$ ))*(Ln(X))	78,96280	86,58555	0,911963	0,3832
(Ln( $\lambda_G + \lambda_p$ ))	-50,39443	48,66669	-1,035501	0,3248
(Ln( $\lambda_G + \lambda_p$ ))^2	0,143238	13,05830	0,010969	0,9915
(Ln( $\lambda_G + \lambda_p$ ))* $\dot{y}$	91,37057	163,1752	0,559954	0,5878
(Ln( $\lambda_G + \lambda_p$ ))*(Ln(X))	-32,92160	44,89008	-0,733383	0,4802
$\dot{y}$	-329,5606	463,7471	-0,710647	0,4935
$\dot{y}$ ^2	-326,2959	558,1015	-0,584653	0,5717
$\dot{y}$ *(Ln(X))	-146,3642	241,2291	-0,606744	0,5575
Ln(X)	186,9378	193,9264	0,963962	0,3578
(Ln(X))^2	42,27903	59,03651	0,716151	0,4903
R-squared	0,431026	Mean dependent var	0,118423	
Adjusted R-squared	-0,365538	S.D. dependent var	0,281414	
S.E. of regression	0,328850	Akaike info criterion	0,897281	
Sum squared resid	1,081425	Schwarz criterion	1,628606	
Log likelihood	3,783992	Hannan-Quinn criter.	1,100119	
F-statistic	0,541106	Durbin-Watson stat	3,241281	
Prob(F-statistic)	0,857581			

Source: Own estimation

### Chart 3: Breusch-Godfrey Serial Correlation LM Test

F-statistic	1,842760	Prob. F(4,16)	0,1700
Obs*R-squared	7,884800	Prob. Chi-Square(4)	0,0959

Test Equation:  
 Dependent Variable: RESID  
 Method: Least Squares  
 Presample missing value lagged residuals set to zero,

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0,556544	2,017992	-0,275791	0,7862
Ln( $\varphi$ )	-0,523521	0,777610	-0,673244	0,5104
Ln( $\lambda_G + \lambda_P$ )	0,063068	0,694692	0,090786	0,9288
$\dot{y}$	-1,092495	3,748661	-0,291436	0,7745
Ln(X)	0,306531	1,480012	0,207114	0,8385
RESID(-1)	-0,246368	0,255592	-0,963914	0,3494
RESID(-2)	-0,448124	0,235270	-1,904724	0,0750
RESID(-3)	-0,502558	0,247110	-2,033744	0,0589
RESID(-4)	-0,219625	0,268227	-0,818800	0,4249
R-squared	0,315392	Mean dependent var	-2,16E-15	
Adjusted R-squared	-0,026912	S.D. dependent var	0,351223	
S.E. of regression	0,355918	Akaike info criterion	1,045480	
Sum squared resid	2,026842	Schwarz criterion	1,484275	
Log likelihood	-4,068500	Hannan-Quinn criter.	1,167183	
F-statistic	0,921380	Durbin-Watson stat	1,932398	
Prob(F-statistic)	0,524469			

Source: Own estimation

## 7.2. Proof that the Optimal is a Maximum

In this section we show that the critical point is always a maximum. We use in this proof the value estimated for  $\alpha_1$  (-4.578) with the OLS regression for the country risk indicator.

First, it can be seen that the second derivative of the optimization function (16) is:

$$2\theta' C_b^{-\sigma} (\delta + \theta + \theta' \varphi) - \sigma C_b^{-1-\sigma} (1-\theta) (\delta + \theta + \theta' \varphi)^2 - C_b^{-\sigma} (1-\theta) (2\theta' + \theta'' \varphi) - \theta'' \frac{C_b^{1-\sigma}}{1-\sigma} + \theta'' \frac{C_d^{1-\sigma}}{1-\sigma} + (1+\Delta q) \left[ 2\theta' C_d^{-\sigma} (1-\delta - \theta - \theta' \varphi) - \sigma C_d^{-1-\sigma} \theta (1+\Delta q) (1-\delta - \theta - \theta' \varphi)^2 - \theta C_d^{-\sigma} (2\theta' + \theta'' \varphi) \right]$$

$\theta' = \theta(1-\theta)$   $\theta'' = \theta(1-\theta) f'$  From the definition of  $\theta$  we obtain that  $\theta' = \theta(1-\theta) f'$  with  $f' = \frac{-4.578}{\varphi}$  and  $\theta'' = \theta(1-\theta) ((f')^2 (1-2\theta) + f'')$  with  $f'' = \frac{4.578}{\varphi^2}$

We assume without loss of generality that  $C_b > C_d$ . This inequality always holds, because during crisis more money is saved and consumption is lower than during normal times. By grouping the terms we obtain the following expressions:

$$\begin{aligned} &\triangleright 2\theta' C_b^{-\sigma} (\delta + \theta + \theta' \varphi) + 2\theta' (1 + \Delta q) C_d^{-\sigma} (1 - \delta - \theta - \theta' \varphi) \\ &= 2\theta' \left\{ (C_b^{-\sigma} - C_d^{-\sigma}) (\delta + \theta + \theta' \varphi) + \Delta q C_d^{-\sigma} (1 - \delta - \theta - \theta' \varphi) + C_d^{-\sigma} \right\} \end{aligned}$$

In addition  $\theta' < 0$ ,  $C_b^{-\sigma} - C_d^{-\sigma} < 0$ ,  $\Delta q \geq 0$ ,  $C_d^{-\sigma} > 0$  and  $\delta + \theta + \theta' \varphi < 1$ . If  $(\delta + \theta + \theta' \varphi) < 0$  then the relation is obviously negative and if  $(\delta + \theta + \theta' \varphi) > 0$  we have that  $(C_b^{-\sigma} - C_d^{-\sigma}) (\delta + \theta + \theta' \varphi) + C_d^{-\sigma} > C_b^{-\sigma} - C_d^{-\sigma} + C_d^{-\sigma} = C_b^{-\sigma} > 0$ . We conclude that the above expression is ever negative.

$$\triangleright -\sigma \left( C_b^{1-\sigma} (1-\theta) (\delta + \theta + \theta' \varphi)^2 + \theta (1 + \Delta q)^2 C_d^{1-\sigma} (1 - \delta - \theta - \theta' \varphi)^2 \right). \text{ This expression is clearly negative considering that } C_b^{1-\sigma}, C_d^{1-\sigma} > 0.$$

$$\triangleright -(1-\theta) C_b^{-\sigma} (2\theta' + \theta'' \varphi) - (1 + \Delta q) \theta C_d^{-\sigma} (2\theta' + \theta'' \varphi). \text{ We observe that } (1-\theta), C_b^{-\sigma}, C_d^{-\sigma}, \theta, (1 + \Delta q) > 0. \text{ As a result the sign of the expression only depends on the sign of } 2\theta' + \theta'' \varphi. \text{ We have that } 2\theta' + \theta'' \varphi = 2\theta(1-\theta) f' + \theta(1-\theta) \left( (f')^2 (1-2\theta) + f'' \right). \text{ If } \theta < 0.5 \text{ the above expression is greater than}$$

$$\theta(1-\theta)(2f' + f'') = \theta(1-\theta) \left( \frac{2^* - 4,578}{\varphi} + \frac{4,578}{\varphi^2} \right) = 4,578 * \theta(1-\theta) \left( \frac{-2\varphi + 1}{\varphi^2} \right)$$

This expression is positive if  $\varphi < 0.5$ . It results the above expression is negative.

$$\triangleright -\theta'' \frac{C_b^{1-\sigma}}{1-\sigma} + \theta'' \frac{C_d^{1-\sigma}}{1-\sigma} = \frac{\theta''}{1-\sigma} \left( -C_b^{1-\sigma} + C_d^{1-\sigma} \right). \text{ As } \theta'' \text{ is always positive if } 1-\sigma \text{ is positive, } -C_b^{1-\sigma} + C_d^{1-\sigma} \text{ is negative and the whole expression is negative, too. In the contrary case, if } 1-\sigma \text{ is negative, } -C_b^{1-\sigma} + C_d^{1-\sigma} \text{ is positive and the whole expression is negative. As a result the above expression is always negative.}$$

After considering all expressions and having shown that all of them are negative, we obtain that the second derivative of the optimization function is negative and therefore the solution to the problem is always a maximum.